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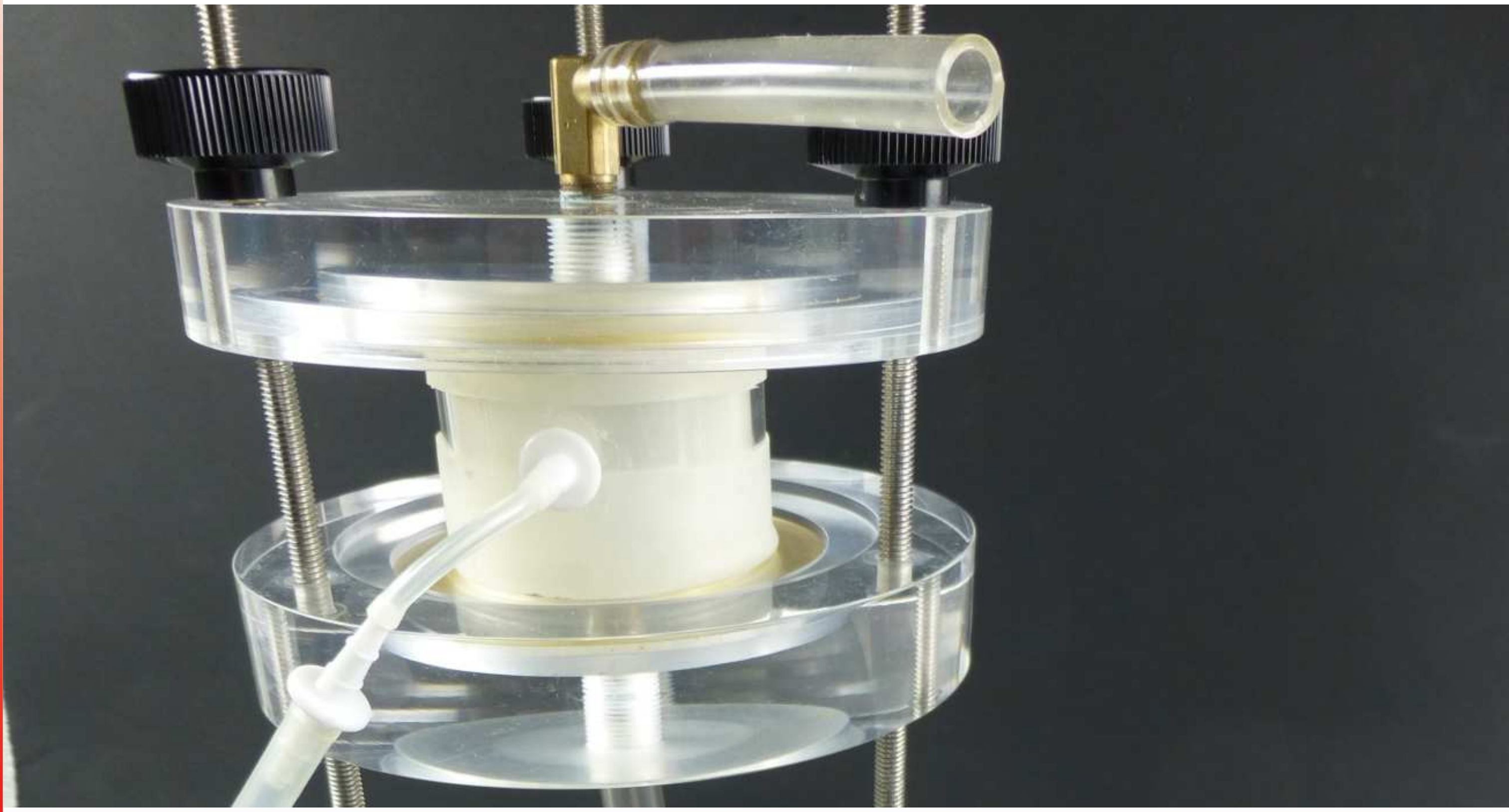

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Standard specimens to determine the hydraulic conductivity of saturated soils

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Summary

Measurement of hydraulic conductivity at saturation (*Ksat*) is prone to errors. Values encountered in soils span more than four orders of magnitude, from > 10^{-3} m/s to < 10^{-7} m/s. Various methods to determine saturated hydraulic conductivity are available, but there are no standard specimens that would allow conductivity measurements to be assessed with respect to absolute accuracy. Only use of custom-made specimens with stable and well-defined conductivity values as conductivity standards will overcome these limitations. To our knowledge, this has never been done, although the concept has long been proposed.

This study designed and tested specimens consisting of cylindrical acrylic glass tubes with an acrylic glass baseplate with one or more holes. Specimen length and outer diameter were identical to those of soil samples typically used for *Ksat* measurements. Specimen conductivity was found to depend on the number and size of holes in the acrylic glass baseplate and on the gradient applied. The theory underlying calculation of conductivity for these specimens is presented. In interlaboratory comparison, different laboratories can run their standard *Ksat* test procedure using the specimens. The specimens also proved sufficiently robust for repeated use and for shipping, permitting their use for comparing different laboratories, methods and laboratory technicians.

Keywords: hydraulic properties, reproducibility, soil sample

1 Introduction

According to Darcy's law, the velocity of macroscopic water flow in saturated soil is proportional to the gradient of the total water potential, with hydraulic conductivity at saturation (*Ksat*) as the proportionality factor. *Ksat* is an important soil parameter for describing water flow and nutrient transport through soil and is extremely sensitive in responding to soil compaction, making it a favorite and hence frequently used parameter for assessing the degree of soil compaction. Therefore, reproducible, accurate measurement of *Ksat* is a matter of importance, although one that has received little research attention to date.

The *Ksat* value can range from more than 10^{-3} m/s (i.e., infiltration of about 3 m per hour) to less than 10^{-7} m/s (infiltration of about 1 cm per day). Due to this wide range of possible values, reproducible, accurate laboratory measurement of *Ksat* remains a challenge. Laboratory methods based on one of two different principles, constant-head or falling-head, are commonly used to determine *Ksat*. There are many versions of both types of method, but there is no standard specimen available that would allow these to be compared with respect to accuracy.

Standard specimens to calibrate *Ksat* measurements are needed particularly for comparison of absolute *Ksat* values measured by different laboratories as part of legal proceedings. *Ksat* values produced within a particular laboratory may be biased, albeit typically in a consistent manner, but comparison of values from different laboratories is worthless without calibrated methods. Thus, only standard specimens will resolve the issue.

In our search for a suitable material to replace soil samples in the calibration of analytical equipment and procedures, we experimented with different porous materials such as sintered glass, sintered steel, sintered brass and metallic mesh. We found that none of these could be reproducibly saturated in a satisfactory manner, as the degree of saturation depended on the previous cycle of saturation and desaturation, the starting point, and the course of the current saturation process. Although these problems related to the saturation process could be resolved, they make it impossible for these materials to be used confidently in standard procedures.

We therefore developed an alternative design for standard specimens (Figure 1). We studied the feasibility of this design first to calibrate a custom-made apparatus for determining *Ksat* (Figure 2) and then to calibrate commercially available apparatus at other laboratories. Our standard specimen consisted of an acrylic glass tube of length *L* with inner diameter *D* and with an acrylic glass baseplate with hole/s of diameter *d* and length *l* corresponding to the thickness of the baseplate. The water flew from below through the hole, which acted as an artificial pore.

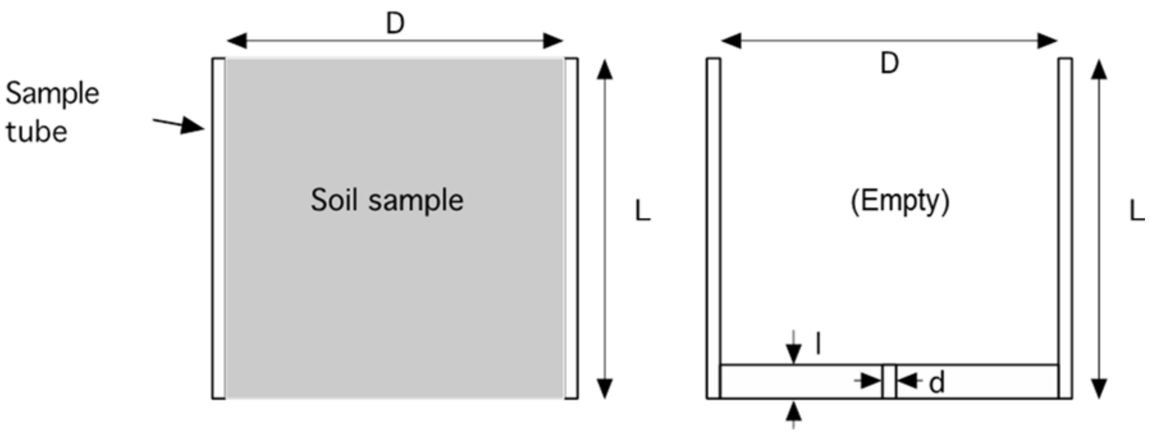

Figure 1 | Soil sample (left) and standard specimen (right). D: diameter of sample, L: length of sample, d: diameter of pore (drilled hole), l: length of pore.

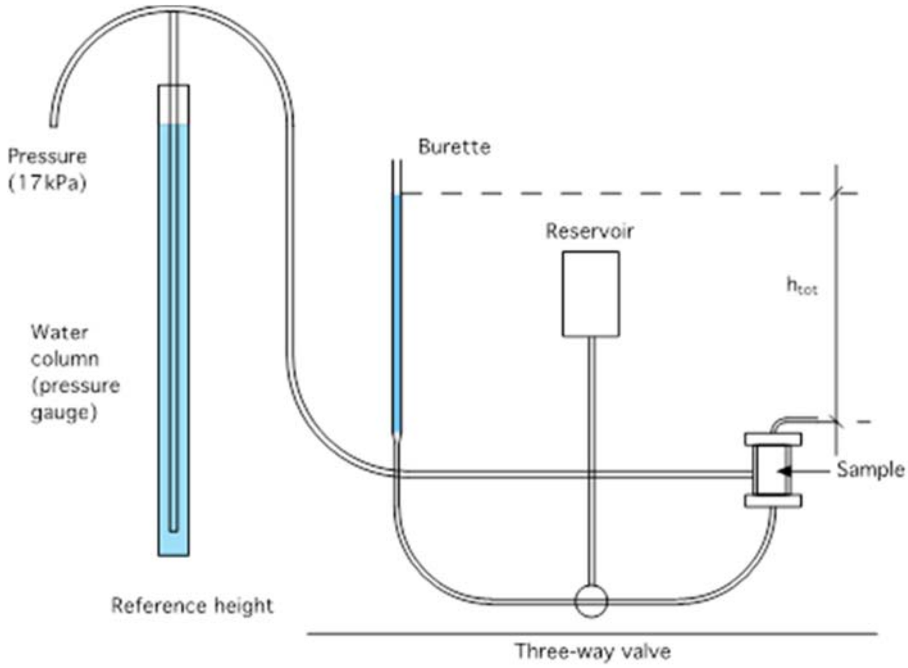

Figure 2 | Apparatus for measuring the hydraulic conductivity at saturation by the falling-pressure head method.

Replacing soil samples with robust standard specimens of known conductivity allows comparison of different laboratories, methods and laboratory technicians.

2 Theoretical considerations

2.1 Basics

Darcy's law relates the volume *Q* per unit time *t* of water flowing through a soil sample of cross-sectional area *A* and length *L* to the pressure head difference *h* as:

$$\frac{Q}{t} = AK\frac{h}{L} = A\,V \qquad (1)$$

where *K* [1] is the hydraulic conductivity at saturation. Water flow volume per unit time is the cross-sectional area of the sample multiplied by the flow velocity *V*.

Hagen-Poiseuille's law gives the flow volume through a cylindrical pore of radius *r* and length *l* as a function of the pressure head difference *h*:

$$\frac{Q}{t} = \frac{\pi r^4 \rho g h}{8\eta l} = \pi r^2 \frac{r^2 \rho g h}{8\eta l} = a\,v \qquad (2)$$

with $$v = c_H \frac{h}{l} \qquad (3)$$

where $$c_H = \frac{r^2 \rho g}{8\eta}$$

and where ρ denotes fluid density, *g* acceleration due to gravity and η the dynamic viscosity of water. Water flow volume per unit time equals the cross-sectional area *a* of the pore multiplied by flow velocity *v* within the pore.

The water volume flowing through the drilled hole of the specimen equals the water volume flowing through the tube of the specimen, because the whole apparatus is saturated and thus there is no change in water content with time. Therefore:

$$\frac{Q}{t} = AV = a\,v \qquad (4)$$

Hagen-Poiseuille's law is based on straight, cylindrical tubes of equal diameter and on Newton's law of viscosity only. Any additional flow resistance leads to a pressure head difference h_c, proportional to v^2 (Glück, 1988), in addition to the effective pressure head difference h_{eff}. Therefore, the total pressure head difference h_{tot} equals:

$$h_{tot} = h_{eff} + h_c = h_{eff} + \frac{1}{2g}\zeta v^2 \qquad (5)$$

where ζ is the overall empirical resistance parameter. The empirical parameter ζ can be estimated using standard methods for calculating flow in ventilation and heating tubes (Glück, 1988; see Appendix). For a step-wise change in tube diameter from 50 mm to 1 mm and back to 50 mm, the value of ζ equals 1.5.

[1] To facilitate the notation, *K* instead of *Ksat* is written.

2.2 Converting total to effective pressure head

Total pressure head difference is commonly used for Darcy's law, whereas the effective pressure head difference has to be used for Hagen-Poiseuille's law. Replacing v^2 in equation (5) with the term for v in equation (3) gives the quadratic equation:

$$\frac{1}{2g}\zeta\frac{c_H^2}{l^2}h_{eff}^2 + h_{eff} - h_{tot} = 0$$

with the following solution:

$$h_{eff} = \frac{\sqrt{1+4ch_{tot}}-1}{2c} \qquad (6)$$

where $\quad c = \frac{1}{2g}\zeta\frac{c_H^2}{l^2}$

The effective hydraulic head is smaller than the total applied hydraulic head. For $h_{tot} \to 0$, $h_{eff}/h_{tof} \to 1$, whereas for $h_{tot} \to \infty$, $h_{eff}/h_{tot} \to 0$. With equation (6), total applied pressure head is converted to effective pressure head when the proposed specimens are used.

2.3 Constant hydraulic pressure head

2.3.1 Calculating hydraulic conductivity

Hydraulic conductivity of soil samples at saturation can be determined by keeping the total hydraulic pressure head constant. To predict the hydraulic conductivity of the specimens, Darcy's law (equation [1] with h_{tot}) can be equalized with Hagen-Poiseuille's law (equation [2] with h_{eff}) and solved for K, yielding:

$$K_1 = c_H\frac{a}{A}\frac{L}{l}\frac{h_e}{h_t} = K_0\frac{h_{eff}}{h_{tot}} \qquad (7)$$

Replacing h_{eff} with (6) gives:

$$K_1 = \mathrm{K}_0\frac{\sqrt{1+4ch_{tot}}-1}{2ch_{tot}} \qquad (8)$$

With equation (8), the hydraulic conductivity K_0 of the specimen can be predicted based on the physical properties of the specimen and of the fluid. The value K_1, which is actually measured, depends on the ratio of effective to total pressure head. If h_{tot} is small, $h_{eff} \approx h_{tot}$, $K_1 \approx K_0$ (e.g., h_{tot} = 0.05 m $\to$ h_{eff} = 0.047 m $\approx$ h_{tot}).

2.3.2 Measuring hydraulic conductivity

Darcy's law (equation [1]) solved for K yields:

$$K_2 = \frac{QL}{tA}\frac{1}{h_{tot}} \qquad (9)$$

With $h_{tot} = h_{eff} = h$ (and hence $h_c = 0$), equation (9) is the common form of Darcy's law used for determining the hydraulic conductivity of soil samples. However, for the standard specimens developed here, h_{tot} in equation (9) has to be replaced by h_{eff} given by equation (6), yielding:

$$K_3 = \frac{QL}{tA}\frac{2c}{\sqrt{1+4ch_{tot}}-1} \qquad (10)$$

Hydraulic conductivity values K_3 calculated with equation (10) are independent of the total pressure head applied and therefore constant.

2.4 Falling hydraulic pressure head

2.4.1 Calculating mean effective pressure head

The hydraulic conductivity of soil samples at saturation can also be determined by the falling-head method, where total hydraulic pressure head is steadily declining. To predict hydraulic conductivity, total applied pressure head given by equation (6) has to be integrated over the range of pressure heads applied as follows:

$$\int_{h_{tot,i}}^{h_{tot,f}} h_{eff,m}\, dh_{tot} = \int_{h_{tot,i}}^{h_{tot,f}} \frac{\sqrt{1+4ch_{tot}}-1}{2c}\, dh_{tot} \qquad (11)$$

where the subscripts *i* and *f* denote initial and final pressure head. Integration yields the mean effective pressure head $h_{eff,m}$ during measurement:

$$h_{eff,m} = (F_0(h_{tot,i}) - F_0(h_{tot,f}))\frac{1}{h_{tot,i}-h_{tot,f}} \qquad (12)$$

with

$$F_0(h_{tot}) = \frac{1}{2c}\left(\frac{1}{6c}(1+4ch_{tot})^{1.5} - h_{tot}\right)$$

2.4.2 Predicting hydraulic conductivity

Equation (8) can be written as:

$$\int_{h_{tot,i}}^{h_{tot,f}} K_4\, dh_{tot} = \int_{h_{tot,i}}^{h_{tot,f}} K_0 \frac{\sqrt{1+4c\,h_{tot}}-1}{2c\,h_{tot}}\, dh_{tot} \qquad (13)$$

Integration then yields the mean hydraulic conductivity K_4 during measurement:

$$K_4 = K_0(F_1(h_{tot,i}) - F_1(h_{tot,f}))\frac{1}{h_{tot,i}-h_{tot,f}} \qquad (14)$$

with

$$F_1(h_{tot}) = \frac{1}{c}\left(\sqrt{1+4c\,h_{tot}} - ln\left(\sqrt{1+4c\,h_{tot}}+1\right)\right)$$

With equation (14), the hydraulic conductivity K_0 of the specimen can be predicted based on the physical properties of the specimen and the fluid. The value K_4, which is actually measured, depends on the ratio of effective to total pressure head. In contrast to the constant-head method, h_{tot} is usually greater and has to be replaced by h_{eff} to obtain *K* (e.g., h_{tot} = 0.5 m → h_{eff} = 0.36 m).

2.4.3 Measuring hydraulic conductivity

With $Q = \Delta h \times b$, Darcy's law (equation [1]) solved for *t* yields:

$$\Delta t = \frac{b\,L}{K\,A}\frac{\Delta h}{h} \qquad (15)$$

where *b* denotes the cross-sectional area of the burette and Δh the difference in pressure head. With h_{tot} = h_{eff} = *h* (and hence h_c = 0), integration of

$$\int_{t_i}^{t_f} dt = \int_{h_i}^{h_f} \frac{b\,L}{K\,A}\frac{dh}{h} \qquad (16)$$

yields

$$K_5 = \frac{b\,L}{t\,A}(ln(h_f) - ln(h_i)) \qquad (17)$$

Equation (17) is the common equation used to measure the hydraulic conductivity of soil samples, where h = h_e = h_t and hence h_c = 0 (Klute and Dirksen, 1986).

However, for the standard specimens developed here, h in equation (17) has to be replaced by h_{eff} and h_{eff} itself by h_{tot}, giving:

$$\int_{t_i}^{t_f} dt = \frac{b\,L}{t\,A} \int_{h_{tot,i}}^{h_{tot,f}} \frac{2\,c}{\sqrt{1+4\,c\,h_{tot}}}\,dh_{tot} \qquad (18)$$

Integration finally yields:

$$K_6 = \frac{b \cdot L}{t \cdot A}\left(F_2(h_f) - F_2(h_i)\right) \qquad (19)$$

with

$$F_2(h_{tot}) = \sqrt{1+4\,c\,h_{tot}} + ln\left|\sqrt{1+4\,c\,h_{tot}} - 1\right|$$

3 Materials and methods

Two specimens of diameter 50 mm and length 100 mm, one with a single pore with diameter 0.5 mm and the other with a single pore with diameter 1 mm, were custom made. They were mounted in the test apparatus (Figure 2), three sets of measurements were made, and specimens were removed. This procedure was repeated three times.

The hydraulic conductivity values of the empty apparatus pK_A (= $-log(K_A)$) and of the apparatus together with each specimen (pK_{A+S}) were determined using equation (17). Next, the hydraulic conductivity of the specimen alone was calculated as the difference in flow resistance. The flow resistance itself was equated to the reciprocal value of the hydraulic conductivity:

$$pK = log_{10}\left(10^{pK_{A+S}} - 10^{pK_A}\right) \qquad (20)$$

Subsequently, the hydraulic conductivity with respect to effective pressure head was calculated using equation (19). Finally, normalization with respect to sample length (1 m) and cross-sectional area (1 m^2) yielded pK^n:

$$pK^n = pK + log(L) - log(A) \qquad (21)$$

4 Results

The values obtained for the two specimens are given in Table 1. For the specimen with one pore of diameter 0.5 mm, the difference between the hydraulic conductivity measured directly and after eliminating the influence of the empty apparatus was negligible, showing the marginal influence of the latter. The difference of 0.46 units between the values before and after correction for effective pressure head indicates the pressure loss. This value is still dependent on the length and diameter of the sample. After normalization (+ $log_{10}(L)$ – $log_{10}(A)$ = – 0.1 + 2.71 = + 1.71), the measured, corrected and normalized hydraulic conductivity value for the specimen with one pore of diameter 0.5 mm differed by less than 0.01 from the corresponding calculated value.

The measured hydraulic conductivity of the second specimen, with one pore of diameter 1 mm, was larger than that of the specimen with one pore of diameter 0.5 mm, but small compared with that of the empty apparatus. The difference between the hydraulic conductivity measured directly and after eliminating the influence of the apparatus was hence slightly larger. The difference between the values before and after correction for effective pressure head was double that obtained for the other specimen, showing the larger pressure loss. According to equation (2), larger pore diameter leads to faster flow (linear dependence), and according to equation (3), larger flow velocity leads to an even larger pressure loss (quadratic dependence). The value of the measured, corrected and normalized hydraulic conductivity differed by 0.03 from the corresponding calculated value.

Table 1 | Hydraulic conductivity ($pK = -log_{10}(K$[m/s])) of two test specimens (Diameter 50 mm; Length 100 mm), measured with the falling-head method (mean of 9 measurements), corrected for effective pressure head and normalized (Length = 1 m, Area = 1 m^2). Predicted values are given in the bottom row.

Measurement		**Diameter of pore [mm]**	
		0.5	**1.0**
	Equation	***pK***	***pK***
Apparatus without specimen	(17)	2.63	2.63
Apparatus with specimen	(17)	4.28	3.63
Specimen alone	(20)	4.27	3.58
Specimen corrected for effective pressure head	(19)	3.81	2.59
Specimen corrected and normalized	(21)	5.52	4.29
Predicted, normalized value	(13), (21)	5.52	4.32

5 Discussion and conclusions

Although the calculated hydraulic conductivity values obtained for the standard specimens were approximations valid only for pore holes with smooth walls and sharp edges, the differences between measured and calculated values were extremely small. In addition, the hydraulic conductivity values determined with different equipment could be normalized with respect to sample length and sample cross-sectional area, allowing comparison between different methods, laboratories and laboratory technicians.

The hydraulic conductivity of soil samples at saturation has been measured for more than half a century, and it is one of the key soil physical parameters—if not the most important parameter—in detecting and demonstrating soil compaction. However, to date no standard sample has been available to assess the quality of laboratory measurements produced by different laboratories, different types of apparatus or different laboratory technicians. This lack of comparability evolved into an unwarranted firm trust of laboratories in their own measurements and in a particular method, be it the constant-head or falling-head type.

The low-cost standard specimens developed here finally meet the requirements for reference samples. They allow measurements of soil hydraulic conductivity at saturation to be calculated. They can also be used as many times as necessary, by different laboratories, in different types of apparatus and by different operators, and can easily be shipped all over the world. Most importantly, the expected value can be calculated fairly accurately in advance.

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List of figures

Appendix

The value of the total empirical coefficient ζ of resistance of a fluid within a tube is the sum of single coefficients ζ_i (Glück, 1988, p. 61, equation 4.6):

$$\zeta = \zeta_1 + \zeta_2$$

The coefficient of our specimen is therefore the sum of the coefficient for the reduction from sample size to pore size, followed by a widening back to the original sample size. It is obtained as follows (Glück, 1988, p. 75, equation 4.56):

$$\zeta_i = \left(1 - \frac{1}{\mu_i}\right)^2$$

For the reduction from a diameter *D* with a cross-sectional area *A* $(= \pi/4.D^2)$ to a diameter *d* with a cross-sectional *a*, μ_i equals (Glück, 1988, p. 75, equation 4.57):

$$\mu_1 = \left(1 + \sqrt{\frac{1}{2} \cdot \left(1 - \frac{a}{A}\right)}\right)^{-1}$$

For the widening back to the original diameter, μ_i equals (Glück, 1988, p. 74, equation 4.54):

$$\mu_2 = \frac{A}{a}$$

This yields the following total empirical coefficient ζ of resistance of the specimen:

$$\zeta = \zeta_1 + \zeta_2 = \frac{1}{2} \cdot \left(1 - \frac{a}{A}\right) + \left(1 - \frac{a}{A}\right)^2$$

For a specimen with diameter 5 cm, the total coefficient of resistance ζ equals 1.4990 in case of a pore diameter of 1 mm and 1.4997 in case of a pore diameter of 0.5 mm. Both may be approximated by a value of 1.5.