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Full Research Article

How did farmers act? Ex-post validation of linear and positive mathematical programming approaches for farm-level models implemented in an agent-based agricultural sector model

Gabriele Mack*, Ali Ferjani, Anke Möhring, Albert von Ow, Stefan Mann

Agroscope, Socioeconomics Research Group, 8356 Ettenhausen, Switzerland

Abstract. This study evaluates linear programming (LP) and positive mathematical programming (PMP) approaches for 3,400 farm-level models implemented in the SWISSland agent-based agricultural sector model. To overcome limitations of PMP regarding the modelling of investment decisions, we further investigated whether the forecasting performance of farm-level models could be improved by applying LP to animal production activities only, where investment in new sectors plays a major role, while applying PMP to crop production activities. The database used is the Swiss Farm Accountancy Data Network. Ex-post evaluation was performed for the period from 2005 to 2012, with the 2003-2005 three-year average as a base year. We found that PMP applied to crop production activities improves the forecasting performance of farm-level models compared to LP. Combining PMP for crop production activities with LP for modelling investment decisions in new livestock sectors improves the forecasting performance compared to PMP for both crop and animal production activities, especially in the medium and long term. For short-term forecasts, PMP for all production activities and PMP combined with LP for animal production activities produce similar results.

Keywords. Agent-based sector model, farm-level model, linear programming, positive mathematical programming, ex-post validation.

JEL codes. C61, Q18, Q19.

1. Introduction

Agricultural policy models apply either linear programming (LP) or positive mathematical programming (PMP) approaches to analyse the impact of policy changes. The main advantages of PMP models over conventional LP models are that they guarantee exact calibration to the base year and avoid predicting overspecialisation without adding weakly justified constraints to the model formulation (Kanellopoulos *et al.*, 2010). Further

*Corresponding author: Gabriele.mack@agroscope.admin.ch

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advantages of PMP models are that they do not require large datasets and can be viewed as a bridge between econometric models, with substantial data requirements, and more limited LP models (Heckelei and Britz, 2005; Howitt *et al.*, 2012).

Studies evaluating the practice of PMP more than 15 years after Howitt published the first paper on this subject in 1995 show that PMP has become very popular in aggregated policy-decision support models (Garnache *et al.*, 2015; Heckelei *et al.*, 2012). The popularity of PMP is underscored by the fact that the majority of both European and non-European aggregated sector models[1] have used it for the calibration of crop and animal production since 2000.

However, PMP is much less popular in farm-level models. One reason for the limited use of PMP in this context is that farm-level models generally only take into account the activities observed during the reference period, even though new policies and market conditions allow farmers to undertake new production activities. To date, only a few farm-level models have used PMP to calibrate the crop activities of arable farms (Iglesias *et al.*, 2008; Kanellopoulos *et al.*, 2010) or both animal and crop production activities of dairy-farm models (Buysse *et al.*, 2007, Louhichi *et al.*, 2010). Iglesias *et al.* (2008) extended the PMP approach by incorporating new irrigation technologies for crop production activities in farm-level models using PMP.

Farm-level models implemented in agent-based models, which use mathematical programming methods to determine the production decisions of the farm agents (Happe, 2004; Röder and Kantelhardt, 2009; Lobianco and Esposti, 2010; Schreinemachers *et al.*, 2011), also prefer an LP approach over PMP. To our knowledge, there have been, to this point, no farm-level models implemented in agent-based models which use PMP.

The aim of this study is to assess the best mathematical programming approach for farm-level models implemented in the SWISSland[2] agent-based agricultural sector model on an empirical basis, i.e. going beyond theoretical considerations. We analysed the forecasting performance of a linear optimisation approach compared to a PMP approach. Because there is no single PMP approach in practice, but several different mathematical versions of PMP which all influence the forecasting performance of farm-level models, this study reviewed the most frequently used approaches for application in single farm models. To overcome limitations of the PMP approach regarding the modelling of investment decisions, we further investigated whether the forecasting performance of farm-level models could be improved by applying LP only to those production activities where investment in new sectors plays a major role. This is why we also validated an approach which combines PMP for crop production activities and LP for animal production activities. The ex-post evaluation was carried out for the period from 2005 to 2012, with the 2003-2005 three-year average as a base year. Over this period, Swiss agricultural policy changed decisively, particularly for milk and meat production. To cite an example, Swit-

[1] Examples of PMP-based, aggregated models representing either farm-type groups or whole regions are the German FARMIS model (Offermann *et al.*, 2005), the Italian FIPIM model (Arfini *et al.*, 2011), the Spanish PROMAPA model (Júdez *et al.*, 2008), the European CAPRI-FARM model (Gocht and Britz, 2011), the Swiss SILAS model (Mann *et al.*, 2003), the German-Austrian Glowa-Danubia Decision-Support System model (Winter, 2005), the European CAPRI-REG model (Britz and Witzke, 2014), the Dutch DRAM model (Helming, 2005), the USDA REAP model (Johansson *et al.*, 2007), the California SWAP model (Howitt *et al.*, 2012) and the New Zealand model NZFARM (Daigneault *et al.*, 2014).
[2] SWISSland' is the German acronym for 'Structural Change Information System Switzerland.

zerland concluded a free-trade agreement for cheese with the EU in 2007. The same year saw the country's gradual withdrawal from the milk quota system (Flury *et al.*, 2005), as well as the introduction of direct payments for dairy cows.

Section 2 of this paper gives a brief overview of an LP approach for single farm optimisation models and describes the most relevant PMP versions considered for the evaluation. Section 3 gives an overview of the SWISSland agent-based sector model and describes the different PMP and LP modelling options tested for the 3,400 single farm models implemented in the SWISSland model for the ex-post period from 2005 to 2012. By drawing a comparison with the historical pathway, Section 4 illustrates the forecasting performance of the single farm models at the farm and sectoral scales, and Section 5 provides conclusions as to how PMP and LP could be used in farm-based modelling.

2. Overview of LP and PMP approaches

Mathematical programming has been used in agricultural economics for more than fifty years. Mathematical programming starts from a decision rule of the decision maker, which determines the levels of the different variables when aiming to optimise the objective set by the decision maker (Hazell and Norton, 1986). Mathematical programming applied to farm models maximises the farm profit.

$$\max Z = \sum_i p_i x_i - c_i x_\mathrm{i} \tag{1a}$$

$$\text{subject to: } \sum_i A_{wi} x_i \leq B_w \text{ and } x_i \geq 0 \tag{1b}$$

In Equation 1a, parameter Z denotes the farm profit to be maximised, p is the vector of product prices, c is the vector of variable costs, x is the vector of production levels and i is the index for the production activities. The optimal solution must fulfil the constraints in Equation 1b, where B_w is the available quantity of resource endowments w, and A is the demand of resource endowments of one unit of x. Mathematical programming models assuming constant marginal costs in the objective function became generally known as LP models. A main disadvantage of LP models is a tendency to overspecialise in crop production (Howitt, 1995). This was the main reason why Howitt (1995) developed models based on the PMP technique. Howitt *et al.* (2012; 245) describe PMP as a 'deductive approach to simulating the effects of policy changes on cropping patterns at the extensive and intensive margins. The term *positive* implies the use of observed data as part of the model calibration process'.

PMP models use information contained in shadow values of an LP model which is bound to observed activity levels by calibration constraints (Step 1). Based on these shadow values, a non-linear objective function is specified such that observed activity levels are reproduced by the optimal solution of the new programming problem without bounds (Step 2).

Many PMP-models use a quadratic, decreasing marginal gross margin function (Equation 2) that assumes increasing marginal costs in the objective function, whilst returns to scale remain constant. This functional form was proposed by Howitt (1995) because of increasing variable costs per unit of production due to inadequate machinery and management capacity, and due to decreasing yields related to land heterogeneity.

$$\max Z = \Sigma_i p_i x_i - d_i x_i - \tfrac{1}{2}\, x_i Q_{ii} x_i \tag{2}$$

$$Q_{ii} = \frac{1}{\rho_{ii}} * \frac{revenue^{*}}{x_i^{*}} \tag{3}$$

$$d_i = c_i - \lambda_i - Q_{ii} x_i^{*} \tag{4}$$

In Equation 2, parameter d_i denotes the vector of the linear term for crop and animal production activity i of the quadratic objective function, whilst Q_{ii} denotes the symmetric, positive (semi-) definite matrix of the quadratic cost term. Most PMP models estimate the matrix coefficients Q_{ii} and d_i of the quadratic cost terms based on exogenous supply elasticities ρ_{ii} from the literature, according to Equation 3. In Equation 3, the parameter *revenue** denotes the observed revenues from product sales in the base year and parameter x_i^{*} denotes the production levels of the base year. To determine the coefficients d_i and Q_{ii} (Equations 3 and 4), the shadow values λ_i of the calibration constraints for both marginal and preferential activities need to be recovered from the primal LP model described in Equation 1. In 'standard' PMP the cost functions are estimated for each production activity x_i separately, whilst Röhm *et al.* (2003) consider the elasticity of substitution among interrelated crops.

Because, in 'standard' PMP, increasing marginal costs are only assumed for preferential activities whilst constant costs are applied for marginal activities, PMP has often been criticised for its arbitrary assumptions (Howitt *et al.*, 2012; Kanellopoulos *et al.*, 2010). Thus, two PMP versions (Howitt *et al.*, 2012) have been developed to overcome these limitations. The first PMP version, the 'extended PMP variant', was published by Kanellopoulos *et al.* (2010). It solves this problem by estimating a Q matrix for either marginal or preferential activities by using exogenous land rents β in the linear objective function for the available area y according to Equation 5:

$$\max Z = \Sigma_i p_i x_i - c_i x_i - \beta * y \tag{5}$$

Another variant of PMP was proposed by Paris and Howitt (1998). This variant estimates the resource and calibration constraint shadow values based on maximum entropy (ME).

3. Methods and database

3.1 Overview of the agent-based sector model SWISSland

The agent-based SWISSland model depicts 3,400 farms from the Swiss Farm Accountancy Data Network [FADN] data pool as realistically as possible in terms of their operational and cost structures, as well as their social behaviour, as a representative sample of the estimated 50,000 family farms in Switzerland. The key objects of the model are agents representing FADN farms. For each farm, we model production and investment decisions, farm takeover and farm exit decisions, as well as lease decisions for land plots and inter-

Table 1. Behavioural and decision submodels included in the SWISSland agent-based sector model and data collection sources.

		Data Collection				Decision Model						
Submodels	Behaviour	Sample survey (FADN)	Sample survey (representative)	Census data	GIS data	Bayesian network	Microeconomic	Heuristic rule-based	Space theory-based	Institution-based	Preference-based	Hypothetical rules
Agent rational decision module	Production decisions	x					x					
Farm manager's life cycle	Farm takeover, Farm exit		x					x			x	
Land market	Lease decisions for land plots			x	x		x	x	x			x
Growth and investment	Investment decisions	x					x					
	Strategy for shifts in labour input	x				x	x	x			x	

action among agents on the land market (Table 1, categorised according to An [2012]). Table 1 also lists the various data sources and the methods we use for modelling the decision-making of the agents.

For the modelling of lease decisions, a spatial structure of representative reference municipalities was implemented in the model (Mack *et al.*, 2013). This allows the farms to interact on the land market. These interactions are only possible within the lease regions and with (constructed) neighbouring agents, however. A lease algorithm enables the plot-by-plot allocation of exiting farms' land to the remaining farms operating in the immediate vicinity. A plot-by-plot bidding process models which neighbouring agent receives the freed-up land and at what lease price. The neighbouring agent achieving the highest expected increase in income with the lease of the plot receives the lease plot.

Exiting farms are those where the farm manager is not passing on the farm to a successor, or where the potential successor decides against farm takeover on economic grounds. Two income parameters, (1) household income per farm and (2) agricultural income per total labour input, were selected to model farm takeover decisions. Income criteria to model farm exits and farm entries were derived from the regional income levels in the previous period from 2005 to 2012.

A detailed description of the different modules of the SWISSland model can be found in Möhring *et al.* (2016). Because this paper focuses on the modelling of production and investment decisions, we present this issue in detail in Section 3.2.

The model simulates a forecast period of up to thirty calendar years, corresponding more or less to a generational cycle of the farming family. The adaptive reactions of the individual agents and their behaviour when interacting with other agents are depicted in annual steps.

SWISSland calculates sectoral output indicators via an extrapolation algorithm. Zimmermann *et al.* (2015) have compared various extrapolation alternatives for the model. Product quantities and prices, land-use and labour trends, income trends according to the Economic Accounts for Agriculture, sectoral input and output factors for calculating environmental impacts, and key structural figures, such as number of farms, size and type of farm or number of farms changing their farming system, are all sectoral output indicators.

3.2 Options for modelling production and investment decisions

Rational agent behaviour is taken as an important basic assumption of the model. Hence, each agent maximises its annual household income for each time period t (Equation 6).

In keeping with the theory of adaptive expectations, the agents (a) make their production decisions based on price (p) and yield (ε) expectations from the previous year (t-1) for the various animal (l) and crop production (g) activities. Prices and yields were estimated for each agent on an individual-farm basis using the FADN data for the base year, with the observed price trends and average annual yield changes (Δ) resulting from 2000 to 2012 being stipulated exogenously for each time period.

Household income results from the sale of agricultural products originating from land use (*LAND g*) and livestock farming (*ANIMAL l*), from off-farm work (*OFFFARM o*), and from the proceeds of direct payments (*PAYMENT d*) less the means-of-production costs (*COSTFUNCTION*). The level of direct payments corresponds to the year-specific, production-dependent and production-independent approaches in each case, in accordance with current agricultural-policy provisions. Because this study tests various linear and PMP-based quadratic cost functions for crop and animal production activities, the cost functions are described in detail in the Equations 7-12 below.

$$Max\ INCOME_{a,t} = \Sigma_g p_{a,g} * \Delta p_{t\text{-}1,g} * \varepsilon_{a,g} * \Delta\varepsilon_{t\text{-}1,g} * LAND_{a,t,g} + \Sigma_l p_{a,l} * \Delta p_{t\text{-}1,l} * \varepsilon_{a,l} * \Delta\varepsilon_{t\text{-}1,l} * ANIMAL_{a,t,l} + \Sigma_o p_{a,o} * \Delta p_{t\text{-}1,o} * OFFFARM_{a,t,o} + \Sigma_d p_{d,a} * \Delta p_{t,d} * PAYMENT_{a,t,d} - OSTFUNCTION_{a,t}$$

subject to

$$\Sigma_g \omega_{a,g,w}^{LAND} * LAND_{a,t,g} \leq Area_{a,t}$$

$$\Sigma_l \omega_{a,l,w}^{\mathrm{ANIMAL}} * ANIMAL_{a,t,l} \leq Places_{a,t}$$

$$\Sigma_f \omega_{a,f,w}^{\mathrm{LABOUR}} * LABOUR_{a,t,f} * LAND_{a,t,g} + LABOUR_{a,t,f} * ANIMAL_{a,t,l} \leq LABOURCAP_{a,t} \quad (6)$$

The resource endowment (ω) of a farm consists of the available area (*Area*), animal places on the farm (*Places*), other capacities limiting animal and crop production (e.g. sugar beet quota, milk quota up to 2007, provisions on the receipt of direct payments), and labour force (*LABOURCAP*).

The use of individual-farm FADN data ensures that various factors influencing the objective-function and production-coefficient matrix are automatically taken into account, allowing the depiction of numerous management options that are typical for Switzerland. The cost and output parameters of the production activities are therefore heterogeneous and influence the agents' decision-making scope.

Five different options for modelling animal and crop production decisions were analysed in this study (Table 2). Option 1 determines both crop and animal production decisions based on linear cost functions for 17 crops and 8 animal production activities according to Equation 7:

$$Max\ INCOME_{a,t} = REVENUE_{a,t} - \Sigma_l c_{l,a} * \Delta c_{t-1,l} * ANIMAL_{a,t,l} - \Sigma_g c_{g,a} * \Delta c_{t-1,g} * LAND_{a,t,g} \quad (7)$$

Option 1 does not calibrate the production activities to base-year levels. It takes into account the uptake of crop production activities which were not observed in the base year, but which occur in the farm's historic crop mix. For animal production activities, it considers the adoption of new production sectors. For modelling new production activities, which were not observed in the base-year, missing information must be added with the help of average values for other farms, or extrapolated using standard data. For all agent activities occurring in the production programme of the forecast years rather than in the base year, the yield and price coefficients are estimated with the aid of a random distribution based on the means and standard deviations of the values for all agents from the same region and farm type (see Möhring *et al.* [2016]).

Options 2a and 2b apply linear cost functions for animal production activities only, while PMP-based quadratic cost functions are used to determine crop production decisions (Equation 8). These options consider only crop production activities which were observed in the base year, whereas, for animal production activities, investment activities in new production sectors are taken into account.

$$Max\ INCOME_{a,t} = REVENUE_{a,t} - \Sigma_g c_{g,a} * \Delta c_{t-1,g} * LAND_{a,t,g} - \Sigma_g d_{a,g} * LAND_{a,t,g} - 0.5\ \Sigma_g Q_{a,g} * LAND^2_{a,t,g} - \Sigma_l c_{l,a} * \Delta c_{t-1,l} * ANIMAL_{a,t,l} \quad (8)$$

Option 2a estimates the matrix coefficients Q of the non-linear cost term based on base-year revenues (*revenue**) and base-year crop production levels (*LAND**), and uses supply elasticities equal to one owing to the lack of empirical data (Equation 9).

$$Q_{g,a} = \frac{revenue^*_{g,a}}{LAND^*_{g,a}} \quad (9)$$

For those production activities where the output is used on the farm itself, is calculated based on linear costs and shadow values according to the German farm type model FARMIS (Schader, 2009):

$$Q_{g,a} = (c_{g,a} + \lambda_{g,a}) \ / \ LAND^*_{g,a} \quad (10)$$

The linear term d of the quadratic cost function is calculated according to Equation 11.

$$d_{g,a} = \lambda_{g,a} - Q_{g,a} LAND^*_{g,a} \quad (11)$$

Option 2b estimates the matrix coefficients of the quadratic cost functions for crop production activities on the basis of maximum entropy. The maximum entropy technique in combination with the PMP calibration allows us to recover a quadratic activity variable cost function accommodating complementarity and substitution relations between activities. To estimate the parameter vector $d_{g,a}$ and the matrix $Q_{g,a}$ of the variable cost support points for the parameters were defined. As a starting point, the linear parameters $d_{g,a}$ could be centred around the observed accounting cost per unit of the activity. For example, the two unknown parameters are specified as an additive function of a number of support points. We could choose five support points Zd (d_1,..d_5) and ZQ (*zq1*,.. *zq5*) for parameter $d_{g,a}$ and the matrix $Q_{g,a}$. The entropy problem is maximised using support-points consisting of a Zd vector and a ZQ matrix. Because no cross cost effects are expected between crop and animal activities, the linear vector d of the quadratic activity cost function is partitioned into one vector which includes the crop activities and a second vector which includes the animal activities. Similarly, the quadratic matrix Q is partitioned into one matrix which includes the crop activities and a second matrix which includes the animal activities. Both PMP approaches guarantee exact calibration of supply decisions at farm and aggregated levels, taking into account the trade of factors among farms. Nevertheless, different approaches can produce different results when used to predict the future behaviour of the farmer.

Options 2a and 2b combine the advantages of both PMP and LP modelling, with PMP calibrating crop production activities to observed base-year levels taking into account the pedoclimatic conditions of the individual farms, and LP enabling modelling of the adoption of new animal production sectors. In all models with a linear cost function in animal husbandry, agents can invest in new barns, allowing them to expand their herd size considerably even within a specific time period, provided that all other necessary resources are available in sufficient quantity. Moreover, switching to new production activities is easily possible in the animal husbandry sector. In order to avoid an objective function with an integer formulation, however, individual barn construction variants (previously selected and evaluated according to plausibility) are tested iteratively with the aid of the loop process for each agent entitled to investment. Here, the annual external costs of the entire building (depreciation, repair, insurance and interest) are taken into account, irrespective of whether the barn can be fully utilised. If the agent is entitled to receive investment credits or investment aid, these lower the interest charges. Ultimately, the variant with the highest positive objective-function value is implemented. In the following year, all animal places resulting from the investment in the barn are available to the farmer. In this case, further use of the old barn is ruled out. Investment activities in new animal sectors are taken into account when a farm successor takes over from his predecessor. Only for older agents it was assumed that investment was primarily in the animal sectors pursued to date.

Options 3a and 3b test PMP-based quadratic production-cost functions for both animal and crop production activities (Equation 12):

$$Max\ INCOME_{a,t} = REVENUE_{a,t} - \Sigma_g c_{g,a} * \Delta c_{t-1,g} * LAND_{a,t,g} - \Sigma_g d_{a,g} * LAND_{a,t,g} - 0.5\ \Sigma_g Q_{a,g} * LAND^2_{a,t,g} - \Sigma_l c_{l,a} * \Delta c_{t-1,l} * ANIMAL_{a,t,l} - \Sigma_l d_{a,l} * ANIMAL_{a,t,l} - 0.5\ \Sigma_l Q_{a,l} * ANIMAL^2_{a,t,l} \quad (12)$$

Because investments in new barns radically alter the cost structure, the PMP-based cost function completely changes the function values derived in the base year. Since no methods were previously available to estimate the change in the PMP-based cost functions derived from the base year, a continuous model approach in which the agents continuously expand their barns by individual animal places was chosen for Options 3a and 3b.

Table 2. Modelling options for determining production and investment decisions in the farm-level models of the SWISSland agent-based sector model.

Option No	Name	Cost function for crop production activities	Cost function for animal production activities	PMP calibration method	Estimate of matrix coefficients of quadratic cost function	Investments
1	Linear	Linear	Linear	-	-	Investment activities for new buildings
2a	Linear-Quad-Revenues	PMP-based quadratic	Linear	Extended	Revenues	Investment activities for new buildings
2b	Linear-Quad-Entropy	PMP-based quadratic	Linear	Extended	Maximum entropy	Investment activities for new buildings
3a	Quad- Revenues	PMP-based quadratic	PMP-based quadratic	Extended	Revenues	Continuous investment costs for buildings
3b	Quad-Entropy	PMP-based quadratic	PMP-based quadratic	Extended	Maximum entropy	Continuous investment costs for buildings

PMP: Positive Mathematical Programming

3.3 Assessing forecasting performance

In this study, we assess the forecasting performance of the options based on the average forecasting error (AFE) measuring the difference between forecasted and historical parameters at the farm and sectoral scales. The farm-scale parameters assess the forecasting performance only of those agents who remained in the sample for the entire simulation period (2005 to 2012). In contrast, sectoral parameters represent changes in the total Swiss farm population over the period from 2005 to 2012 and take into account the farm sample changes due to farm exits and entries. Therefore, the simulation results from all agents were extrapolated to the sectoral scale based on Zimmermann *et al.* (2015).

At the farm scale, the AFE measures the percentage difference between historical and forecasted average production levels for each activity. The weighted average forecasting error (WAFE) of crops aggregates the AFE of all crops based on average production share in the FADN farm sample. The WAFE is calculated analogously for animals. Finally, the total weighted average annual forecasting error (TWAFE) aggregates the WAFE for crops and animals equally. At the farm scale, average crop and animal production levels from all FADN farms over a period of three years represent historical parameters.

At the sectoral scale, we calculate the production changes from 2003-2005 and 2010-2012 in percent. The forecasting error measures the deviation from historical values. At sectoral scale, historical values are based on production changes in the total Swiss farm population over this period.

4. Results

The SWISSland results were obtained for each specification rule of the cost function. Table 3 presents the historical average production levels of the corresponding FADN-farms and the AFE for crop and animal production activities in the short and long term. Linear cost functions for both crop and animal production activities (Option 1) lead at farm scale to the WAFE of almost 50% for crops and to the TWAFE for both animal and crop production in both time periods (Table 3). The results in Table 3 also show that crop activities supported by direct payments, such as extensive grassland, fallow land, oilseed rape, soya and sunflower, are highly overestimated in the linear version (Option 1), whilst PMP for crop production activities significantly reduces the AFE in both time periods. In the short term, the approaches with quadratic production costs for crop activities and linear production costs for animal activities (Options 2a and 2b) show, on average, the same WAFE as Options 3a and 3b with quadratic production costs for both animal and crop production activities. However, in the long term, Options 2a and 2b show better forecasting performance than Options 3a and 3b. The forecasting performance of Options 2a and 2b improves, in particular, for the livestock categories of cattle, dairy cows, suckler cows, horses and hens, which showed above-average production increases from 2005 to 2012 due to investment activities. Furthermore, the AFE of fodder and grassland activities decreases in Options 2a and 2b because these activities are highly influenced by the cattle production level. Only for marginal animal activities, such as sheep and goats, which are underrepresented in the Swiss FADN farm sample, is the AFE higher in the linear version than in the PMP variants. For crop activities as a whole, the entropy versions and the revenue versions lead to similar results in the short and long term. The results also show that both PMP variants (based on revenues or entropy) do not influence forecasting performance where PMP is combined with LP. Where PMP is used for both production categories, the entropy method leads to slightly better forecasting performance in the long term.

Table 4 shows that all model options using PMP (Options 2a to 3b) reproduce the observed farm exits in the long term much better than the linear version (Option 1), which significantly underestimates farm exits. Because high farm income reduces the probability of a farm exit, these results indicate that the linear version (Option 1) significantly overestimates farm specialisation and farm income. Comparing the extrapolated production changes of all agents with the historical production changes in the agricultural sector shows that the options with linear cost functions for animals (Options 2a and 2b) lead to better results in the long term, particularly in the sectors where the highest production increases were previously observed, such as suckler cows, hens, horses, goats and poultry. In these animal sectors, above-average investments in new housing, which overcompensate for the reduced production owing to farm exits, were observed in the past.

Table 3. Short- and long-term results at farm scale: Historical crop and animal production levels of all Swiss FADN farms and forecasting errors of the modelling options.

	Historical parameters Average production levels of all FADN farms			Average forecasting error of modelling options [AFE in %]									
	2003-2005	2006-2008	2010-2012	No 1 Linear[§]		No 2a Linear-Quad-Revenues[‡]		No 2b Linear-Quad-Entropy[†]		No 3a Quad-Revenues[¶]		No 3b Quad-Entropy[¤]	
	Base year	S	L	S	L	S	L	S	L	S	L	S	L
UNIT	(ha)	(ha)	(ha)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)
Bread grain	1.39	1.40	1.46	50	52	10	13	8	12	8	10	6	10
Feed grain	1.07	1.13	0.92	84	80	14	5	14	6	12	12	9	12
Grain maize	0.22	0.20	0.21	28	21	8	2	8	2	4	6	11	4
Silage maize	0.88	0.91	1.00	9	17	7	3	7	3	2	6	3	6
Sugar beet	0.30	0.32	0.32	11	12	3	4	3	4	2	3	3	3
Potatoes	0.30	0.27	0.25	299	326	2	4	2	5	13	8	3	9
Oilseed rape	0.21	0.23	0.30	237	162	14	34	14	33	12	33	12	32
Sunflower	0.04	0.05	0.04	321	444	11	15	11	15	15	15	11	15
Legumes	0.07	0.08	0.05	130	236	19	18	18	19	12	24	15	24
Vegetables	0.09	0.10	0.11	237	224	13	16	13	16	11	15	12	15
Fallow land	0.04	0.05	0.03	73	148	13	25	15	23	2	29	14	24
Temporary grassland	2.86	2.92	3.34	10	22	5	8	3	10	1	11	0	13
Extensive grassland	1.30	1.30	1.31	68	64	1	1	3	5	20	1	3	5
Less-intens. grassland	0.69	0.68	0.65	8	13	16	21	17	23	2	25	18	24
Intensive grassland	8.66	8.79	8.92	9	11	1	2	0	2	14	3	1	2
Extensive pastures	0.21	0.25	0.25	20	21	11	13	7	9	3	16	8	9
Intensive pastures	1.77	1.78	1.69	2	3	2	3	5	0	3	2	6	1
UNIT	(LU)	(LU)	(LU)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)
Livestock (total)	26.98	27.65	29.83	10	13	5	5	4	6	5	11	5	13
Cattle (total)	21.60	22.07	24.00	13	16	7	8	6	8	7	14	8	14
Dairy cows	14.80	14.97	16.21	11	15	6	6	3	6	4	11	4	11
Suckler cows	1.28	1.57	1.86	15	5	1	2	7	5	22	34	22	35
Horses	0.19	0.22	0.20	16	2	1	11	10	2	6	27	4	34
Sheep	0.21	0.22	0.22	14	33	29	33	14	30	4	7	3	8
Goats	0.05	0.05	0.06	8	6	11	10	7	11	5	23	2	23
Sows	3.98	4.14	4.08	5	7	9	10	7	10	6	9	5	7
Fattening pigs	2.52	2.61	2.67	7	1	2	4	8	3	14	13	14	2
Hens	0.34	0.35	0.59	1	25	19	18	0	21	1	40	3	39
Poultry	0.60	0.60	0.67	1	11	13	23	6	12	0	10	0	12

	Historical parameters Average production levels of all FADN farms			Average forecasting error of modelling options [AFE in %]									
	2003-2005	2006-2008	2010-2012	No 1 Linear[§]		No 2a Linear-Quad-Revenues[‡]		No 2b Linear-Quad-Entropy[†]		No 3a Quad-Revenues[¶]		No 3b Quad-Entropy[¤]	
	Base year	S	L	S	L	S	L	S	L	S	L	S	L
				Weighted average forecasting error [WAFE in %]									
Crop production				50	55	4	5	4	5	4	7	4	7
Animal production				10	14	6	10	6	10	7	13	7	11
				Total weighted average forecasting error [TWAFE in %]									
Average				30	34	5	8	5	8	5	10	5	9

S = Short term; L = long term; LU: Livestock Unit; FADN: Swiss Farm Accountancy Data Network Data Pool;
[§] Linear = Linear cost functions for crop and animal production activities;
[‡] Linear-Quad-Revenues = Linear cost functions for animal production activities and PMP-based quadratic cost functions for crop production activities. Estimate of PMP coefficients based on revenues;
[†] Linear-Quad-Entropy = Linear cost functions for animal production activities and PMP-based quadratic cost functions for crop production activities. Estimate of PMP coefficients based on maximum entropy;
[¶] Quad-Revenues = PMP-based quadratic cost functions for animal and crop production activities. Estimate of PMP coefficients based on revenues;
[¤] Quad-Entropy = PMP-based quadratic cost functions for animal and crop production activities. Estimate of PMP coefficients based on maximum entropy.

The results show that modelling investment decisions in new animal capacities based on linear cost functions (Options 2a and 2b) leads to better results than using continuous investment activities combined with quadratic cost functions (Options 3a and 3b). The results also show that PMP used for crop production activities underestimates production increases which are above-average (such as rapeseed, sugar beet, field vegetables etc.). These results are caused by two characteristics of PMP. On the one hand, the farm-level models only take into account the activities observed during the 2005 reference period, so the adoption of new crop production activities in subsequent years could not be taken into account. On the other hand, the quadratic cost functions prevent overspecialisation and above-average production increases for single activities. We can only assess the performance of the model based on its forecasting capacity.

5. Conclusions

This ex-post validation at farm scale clearly shows that, in the short term, supply curve specifications based on PMP only or on PMP combined with LP for selected

Table 4. Long-term results at sectoral scale: Historical sectoral production changes from base year 2003/05 to 2010/12 and deviation of model results from historical sectoral changes (+/- %) of the modelling options.

	Unit	Observed sectoral change from 2003/05 - 2010/12	No 1 Linear[§]	No 2a Linear-Quad-Revenues[‡]	No 2b Linear-Quad-Entropy[†]	No 3a Quad-Revenues[¶]	No 3b Quad-Entropy[¤]
		Historical change (+/-%)	Deviation from historical sectoral change (+/-%) of the modelling options				
Farm exits							
Total farms	Qty.	-11%	5%	1%	0%	2%	3%
Valley region	Qty.	-12%	5%	4%	2%	4%	6%
Hill region	Qty.	-9%	4%	-3%	-4%	0%	-1%
Mountain region	Qty.	-10%	3%	1%	1%	1%	1%
Farm size < 20 ha	Qty.	-18%	0%	-3%	1%	-1%	6%
Farm size 20-30 ha	Qty.	+4	8%	8%	-4%	6%	-2%
Farm size > 30 ha	Qty.	+15%	9%	-4%	-13%	-4%	-19%
Crop production							
Bread grain	ha	-4%	-17%	-11%	-14%	-1%	-3%
Fodder crop	ha	-17%	-36%	-2%	-5%	12%	9%
Potatoes	ha	-17%	28%	-7%	-7%	-1%	1%
Rapeseed	ha	35%	-52%	-48%	-50%	-41%	-43%
Sunflower	ha	-32%	23%	12%	12%	18%	15%
Field vegetables	ha	11%	173%	-14%	-17%	-9%	-10%
Silage maize	ha	12%	2%	-6%	-5%	-14%	-6%
Sugar beet	ha	6%	-23%	-12%	-12%	-11%	-7%
Open arable land	ha	-6%	8%	-6%	-8%	0%	1%
Temporary ley	ha	9%	2%	-3%	-7%	-15%	-13%
Total arable area	ha	-2%	5%	-4%	-7%	-4%	-3%
Permanent grassland	ha	-2%	5%	2%	-2%	3%	-2%
Total utilised agricultural area	ha	-2%	5%	0%	-3%	1%	-2%
Total livestock	LU	3%	1%	-3%	-5%	-9%	-11%
Dairy cows	LU	-6%	4%	2%	1%	-3%	-2%
Suckler cows	LU	55%	-6%	-17%	-20%	-60%	-60%
Pigs	LU	-3%	-1%	-6%	-6%	-3%	-34%
Fattening calves	LU	-13%	2%	1%	2%	12%	24%
Fattening bulls	LU	-6%	14%	5%	2%	2%	2%
Cattle total	LU	2%	0%	-4%	-4%	-11%	-9%
Sheep	LU	-1%	-19%	-21%	-21%	-5%	-3%
Goats	LU	25%	78%	78%	78%	-39%	-42%
Horses	LU	13%	93%	81%	-11%	88%	122%
Broilers	LU	31%	18%	13%	8%	-38%	-40%
Hens	LU	19%	10%	5%	2%	-20%	-20%

Unit	Observed sectoral change from 2003/05 - 2010/12	No 1 Linear§	No 2a Linear-Quad-Revenues‡	No 2b Linear-Quad-Entropy†	No 3a Quad-Revenues¶	No 3b Quad-Entropy¤
	Historical change (+/-%)	Deviation from historical sectoral change (+/-%) of the modelling options				
		Average of absolute deviation from historical sectoral change (%)				
All attributes		20%	12%	10%	13%	16%

LU: Livestock Unit; § Linear = Linear cost functions for crop and animal production activities;
‡ Linear-Quad-Revenues = Linear cost functions for animal production activities and PMP-based quadratic cost functions for crop production activities. Estimate of PMP coefficients based on revenues;
† Linear-Quad-Entropy = Linear cost functions for animal production activities and PMP-based quadratic cost functions for crop production activities. Estimate of PMP coefficients based on maximum entropy;
¶ Quad-Revenues = PMP-based quadratic cost functions for animal and crop production activities. Estimate of PMP coefficients based on revenues;
¤ Quad-Entropy = PMP-based quadratic cost functions for animal and crop production activities. Estimate of PMP coefficients based on maximum entropy.

production activities significantly improve the forecasting performance of an agent-based model compared with specifications based on LP only. For short-term forecasts, where investment decisions do not play a major role, PMP for all production activities and PMP combined with LP produce similar results. For long-term forecasts, the results at farm scale and at sectoral scale show that combining LP for animal production activities with PMP for crop production activities leads to the best forecasting performance. The combined approach could mitigate some limitations of PMP which are relevant mainly in the medium and long term, such as the adoption of new production activities, while still exploiting the advantages of PMP in order to avoid overspecialised model results.

This study confirms also the finding of Buysse *et al.* (2007) that, in sectors where new production activities are expected to be adopted owing to market and policy changes (i.e. switching from direct payments towards market support or opening borders of an isolated country), the LP approach could represent an appropriate solution, in particular in long-term forecasts, whereas, in the case of minor policy changes or in the short term (i.e. slight modifications of direct payments or tariffs), PMP could improve the forecasting results. The underlying reason for this might lie in the fact that farmers have to take both gradual and binary decisions. In animal production, either a new house will be built or it will not. After a radical reform of agricultural policy, the farming business will be continued or not. Our results have shown that, for such binary decisions, LP is effective. For situations where price fluctuations suggest an increase in potatoes at the expense of a farmer's wheat acreage, PMP is a more suitable instrument.

The results show that supply curve specifications based on the extended variant of PMP and that revenues and specifications based on PMP and maximum entropy lead to similar results. The results support other studies by Gocht (2005) and Winter (2005), both of whom discovered that the different PMP versions led to similar model results. Although all tested approaches lead to deviations in the actual observable trends, we may conclude that PMP for crop production activities combined with LP for animal production activities is preferable to full PMP when assessing the forecasting performance of sectoral production changes in the medium or long term.

At the same time, this paper shows that, in general, an ex-post validation makes a valuable contribution to improving the accuracy of the model, but can also make a theoretical contribution to the methods used. On the other hand, this example demonstrates that PMP and LP approaches have their strengths and weaknesses in individual areas. For this reason, the methodological considerations for improving mathematical programming should be continued. This will not only improve the predictive accuracy of the model results, but, just as importantly, it will also have positive consequences for the acceptance of model simulations for use in policy advice.

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